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The Core of the Non-Universe in Jaina Cosmology as a Cube of Eight Space-Points

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1 INTRODUCTION

This paper is not intended as a strong assertion about the history of Jaina thought; rather, it presents a conjectural explanation for understanding the core of the non-universe as a cube of eight space-points and invites further discussion and debate.

According to Jaina cosmology, there is the core of the horizontal universe where a cube of eight space-points is and ten directions originate from those eight space-points. Such a concept is not found outside India.

Walther Schubring wrote that,

> there are 8 centremost units, which, ... are supposed to exist in the centre of the upper regions of the world, called the *Ruyaga*, in Rayanappabhā, ...[1]

J. C. Sikdar wrote that,

> in the Jambūdvīpa island, just in the middle part of the Mandāra-mountain [i.e., Mandara-mountain] ..., here is located the centre of the horizontal region called *Rucaka*, having eight parts [i.e., points], because from this [i.e., these] point[s] flow the ten directions – East, East-South, etc.[2]

1 Schubring 1962: 130. Glossary according to Schubring is as follows. Centremost units (*majjha-paesa*). Glossary is as follows. World (universe), *ruyaga* (Skt. *rucaka*, cube), and Rayanappabhā (Skt. Ratnaprabhā).

2 Sikdar 1964: 521. Glossary is as follows. Region (universe).

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Jozef Deleu wrote that, "the centre of the central, or horizontal, world is the so-called Cube of Eight Space-units ..."[3] M. R. Gelra referred to the cube of eight space-points as the eight-point centre of the universe.[4] Samani Chaitanya Pragya wrote that, "in reference to the central point of the universe, the concept of *Rucaka* is really astonishing and noticeable." She also called it "the centre of the middle section."[5]

These scholars have been largely successful in bringing this important concept of the cube of eight space-points to scholarly attention, on the basis of the texts of the *Ṭhāṇa* (Skt. *Sthānāṅga* [*Sūtra*]) and the *Bhagavaī* (Skt. *Bhagavatī* [*Sūtra*], "The Venerable"). Despite this, it is yet to be understood how there is the core of the horizontal universe where a cube of eight space-points is.

The *Sthānāṅga Sūtra* and the *Bhagavatī Sūtra* are the third and fifth of the twelve books of the Śvetāmbara Jaina canon respectively.[6] Both of them are composed in Arddha-Māgadhī Prakrit. Each of them is an encyclopaedic work. On the basis of mathematical ideas[7] contained in them, they belong to the canonical class of the Jaina school of Indian mathematics.[8]

The *Sthānāṅga Sūtra* is subdivided into ten chapters.[9] The old genuine name of the *Bhagavatī Sūtra* is the *Viyāhapaṇṇatti* (Skt. *Vyākhyāprajñapti*, "Excellent Knowledge about the Explanations" or "Suggestion on the Explanations").[10] *Bhagavaī* (Skt. *Bhagavatī*) is only an epithet which was originally added to the *Viyāhapaṇṇatti* but the former has later on superseded the latter.[11] It is divided into forty-one chapters. Each chapter has sub-divisions called lessons.[12] For a detailed description of the *Jambūdvīpa* Island, the *Bhagavatī Sūtra* refers to the fifth auxiliary canonical book *Jaṃbuddivapaṇṇattī* (Skt. *Jambūdvīpa Prajñapti* [*Sūtra*]).[13]

3 Deleu 1970: 194–195. Glossary according to Deleu is as follows. Horizontal world (*tiriya-loga*), and cube of eight space-units (*aṭṭha-paesiya ruyaga*).

4 Gelra 1999: 121–130.

5 Pragya 2005: 24–25, especially see p. 24. Glossary is as follows. Middle section (horizontal universe).

6 Glossary is as follows. Twelve books (*dvādaśāṅgī*). Each book (*aṅga*) is called a *sutta* (Skt. *sūtra*). According some scholars, there are eleven books because the twelfth book *Ditthivada* (Skt. *Dṛṣṭivāda*) did not survive. See Wiles 2006: 62.

7 Rajgopal 1991: 1–6; Shah 2008: 3–24; Jadhav 2009: 35–53; 2022: 87–102.

8 Jadhav 2017: 321–322.

9 Krümpelmann 2006: 1. Glossary is as follows. Chapter (*ṭhāṇa*, Skt. *sthāna*).

10 At some places it is also written *Viyāhapannatti* or *Vivāhapannatti* or *Viāhapaṇṇattī*. For *Viyāhapannatti* and *Vivāhapannatti*, see Deleu 1970: 17. For *Viāhapaṇṇattī*, see *BhaSū*$_7$: i, iii.

11 Deleu 1970: 17

12 Glossary is as follows. Chapter (*saya*, Skt. *śataka*), and lesson (*uddeso*, Skt. *uddeśaka*). Here the term *saya* (Skt. *śataka*) has no relevance with the number hundred. It can be said to indicate a great number of different teachings gathered in each chapter. See Schubring 1962: 88.

13 *BhaSū*$_3$: 9.1.3, p. 307. Glossary is as follows. Auxiliary canonical book (*uvāṃga*, Skt. *upāṅga*).

These books have come down to us in their present form through three stages of their development. They were first derived from the initial core of the teachings which are attributed to Lord Mahāvīra (599–527 BCE), the twenty-fourth and last *Tīrthaṅkara* in the history of ecclesiastic Jainism, and then given a shape of book by the subsequent individuals. In the second stage, they were collected and fixed in the first synod of monks. According to the *Āvassaya-cuṇṇi* (Skt. *Āvaśyaka-cūrṇi*), generally dated between 593 CE and 693 CE, this synod was held at Pāṭaliputra. According to Hemacandra's *Pariśiṣṭaparvan*, written between 1159 CE and 1172 CE, it was held after 160 years after Lord Mahāvīra. In the third and last stage, they were redacted in the synod of monks held at Valabhī after 980 or 993 years after Lord Mahāvīra or in the fifth century CE.[14]

Geometrical forms are considered in chapter twenty-five of the *Bhagavatī Sūtra* by way of points. The least significant number of points required to construct them are referred to in it. Distinction has been drawn between even and odd number of points. In my paper published in 2009,[15] I showed how those least even and odd numbers were arrived at and identified them as figurate numbers. Those forms with their even and odd number of points are line (2, 3), isosceles triangle (6, 3), square (4, 9), rectangle (6, 15), circle (12, 5), outer-lying circular-annulus (20), pyramid of isosceles triangular base (4, 35), cube (8, 27), cuboid (12, 45), sphere (32, 7), and outer-lying circular-cylinder (40).[16] That paper is highly relevant here. Since how there is the core of the horizontal universe where a cube of eight space-points is is not expressed in the above sources, an explanation is needed to understand it. Therefore, the purpose of this paper is mainly to suggest an explanation to understand it. The concept of the cube is important in that it has a mathematical basis, which I will show in this paper. Based on the information given in the *Bhagavatī Sūtra* and using the concepts of core-circle and core-sphere, I will show how there is the core of the horizontal universe where the cube of eight space-points is. Beyond this, I will explain those ten directions in mathematical terms and discuss other issues related to all these.

This paper is an extension to that paper of mine in the sense that it is only after that paper that I find myself in a position to suggest the aforesaid explanation in this paper and is a partial addendum to that paper in the sense that I will suggest expressions in this paper for the total number of points required to construct the n^{th} core-circle and n^{th} core-sphere, which I could not suggest in that paper.

14 Wiles 2006: 61–63, 75–76; Sikdar 1964: 34; S. K. Jain 1975: 33. Etymologically, "*Tīrthaṅkara*" means "Builder of a ford [across an ocean of suffering]."

15 Jadhav 2009.

16 *BhaSū*$_9$: 25.3.726, pp. 405–407; Jadhav 2009: 38, 42–51.

2 MENTION OF THE CUBE OF EIGHT SPACE-POINTS IN THE BHAGAVATĪ SŪTRA AND STHĀNĀṄGA SŪTRA

In a reply to the question as to where the core of the horizontal universe is, the *Bhagavatī Sūtra* states the following:[17]

> जंबुद्दीवे दीवे मंदरस्स पव्वयस्स बहुमज्झहदेसभाए इमीसे रयणप्पभाए पुढवीए उवरिमहेट्ठिल्लेसु खुड्डगपयरेसु एत्थ णं तिरियलोगस्स मज्झे अट्ठपएसिए रुयए पण्णत्ते, जओ णं इमाओ दस दिसाओ पवहंति, तं जहा— पुरच्छिमा पुरच्छिम—दाहिणा एवं जहा दसमसए णामधेज्जं ति।[18]
>
> "[Amid] the upper and nether thin planes situated in [the top of] this Rayaṇappabhā hell, in the very core [in the centre of the bottom-surface] of Mount Mandara of Jaṃbuddīva Island, is the core of the horizontal-universe; that is where a [cube, namely,] *ruyae* of eight space-points is. From this cube originate the ten directions, namely, east, east-south, etc. Their names can be known from the [first lesson of the] tenth chapter [of the *Bhagavatī Sūtra*]."[19]

Commenting on this passage, Abhayadeva Sūri (eleventh century CE) wrote that at the core of the horizontal universe is a cube of eight space-points contained in two thin planes; one plane is upper and the other is lower.[20] Almost the same passage is given in the *Sthānāṅga Sūtra*.[21] Commenting on that passage, Abhayadeva compared each of the upper and lower sets of the four space-points of this cube of eight space-points with the four teats of a cow's udder, which are one above the other and facing each other.[22]

17 *BhaSū*$_6$: 13.4.13 (question), p. 45. Glossary is as follows. Core (*āyāmamajjha*, Skt. *āyāmamadhya*), and horizontal universe (*tiriyaloga*, Skt. *tiryakloka*).

18 *BhaSū*$_6$: 13.4.13 (reply), p. 45.

19 Glossary is as follows. Upper (*uvarima*, Skt. *uparitana*), nether (*heṭṭhilla*, Skt. *adhastana*), thin (*khuḍḍaga*, Skt. *kṣullaka*, with no thickness), planes (*payaras*, Skt. *prataras*), Rayaṇappabhā (Skt. Ratnaprabhā, Gem-hued), hell (*puḍhavi*, Skt. *pṛthavī*), very core (*bahu-majjha-desa-bhāe*, Skt. *bahu-maddhya-deśa-bhāga*), Mount Mandara (Maṃdara *pavvaya*, Skt. Meru *parvata*), Jaṃbuddīva Island (Jaṃbuddīva *dīva*, Skt. Jambūdvīpa *dvīpa*), core (*majjha*, Skt. *madhya*), cube (*ruyae* or *ruyaga*, Skt. *rucaka*), eight space-points (*aṭṭha-paesas* Skt. *aṣṭa-pradeśas*), east (*puracchimā*, Skt. *pūrva*), east-south (*puracchima-dāhiṇā*, Skt. *pūrva-dakṣiṇa*), and tenth chapter (*dasamasae*, Skt. *daśama-śataka*). For "that is where the cube of eight space-points is," especially see *BhaSū*$_7$: 130.

20 *BhaSū*$_8$: 562. Glossary is as follows. The core of the horizontal universe (*tiryaglokamadhya*), cube (*rucaka*), eight space-points (*aṣṭapradeśa*), two planes (*prataradvaya*), thin (*kṣullaka*), upper (*uparima*), and lower (*adhastana*).

21 जंबुद्दीवे दीवे मंदरस्स पव्वतस्स बहुमज्झदेसभागे इमीसे रयणप्पभाते पुढवीते उवरिमहेट्ठिल्लेसु खुड्डगपतरेसु एत्थ णमट्ठपतेसिते रुयगे पण्णत्ते जओ णमिइमातो दस दिसातो पवहंति, तंजहा— पुरत्थिमा, पुरत्थिमदाहिणा, दाहिणा, दाहिणपच्चत्थिमा, पच्चत्थिमा, पच्चत्थिमुत्तरा, उत्तरा, उत्तरपुरत्थिमा, उड्ढा, अधा। *SthāSū*$_1$: ch. 10, sūtra 720, p. 823. Also see *SthāSū*$_2$: ch. 10, sūtra 30, p. 484.

22 *SthāSū*$_1$: ch. 10 commentary, p. 826, Glossary is as follows. Teats of a cow's udder (*gostana*).

3 RESULTS AND DISCUSSION

ACCORDING TO THE BHAGAVATĪ SŪTRA, aether, anti-aether, space, soul, and matter are five realities or fundamental entities. Time too, in addition to these five realities, appears as a reality in it.[23]

THE UNIVERSE-SPACE AND NON-UNIVERSE-SPACE AND THEIR SHAPES

Space, the third reality, is responsible to provide accommodation. An indivisible part of space is called space-point.[24] We can simply call it point. Space is comprised of infinite such points. It establishes the receptacles of living and non-living substances.[25] It is not entirely occupied by the other realities. Instead, it is partly occupied and partly unoccupied even though it is a homogeneous continuum, ubiquitous, and all-pervading.[26] The part of space which is pervaded by the other realities is called the universe-space and the rest of the space, which is empty, is called the non-universe-space.[27]

The non-universe is all pure infinite space beyond and exists all round the universe. It equals the whole space minus its infinite part, i.e., minus the universe-space.[28] At its central part, there exists the universe. The non-universe has the shape of a hollow sphere.[29]

The universe is limited.[30] The number of space-points in it is innumerable.[31] The universe is described using metaphors. It is like a firmly standing formation. It is wide at the base with a gradual taper ending with the shape of an upright drum. It is divided into three regions, namely, the lower universe, the horizontal

23 *BhaSū*$_6$: 13.4.17, p. 47; *BhaSū*$_7$: 13.4.55, p. 132; Sikdar 1964: 559. Glossary is as follows. Aether (*dhamma*, Skt. *dharma*, medium of motion), anti-aether (*ahamma*, Skt. *adharma*, medium of rest), space (*āgāsa*, Skt. *ākāśa*), soul (*jīva*), matter (*poggala*, Skt. *pudgala*), and time (*kāla*).

24 Deleu 1970: 94. Glossary is as follows. Accommodation (*avagāhaṇā*, Skt. *avagāhanā*), and space-point (*paesa*, Skt. *pradeśa*).

25 *BhaSū*$_6$: 13.4.20, p. 48. Glossary is as follows. Receptacle (*bhāyaṇa*, Skt. *bhājana*), living (*jīva*), non-living (*ajīva*), and substances (*davvas*, Skt. *dravyas*).

26 Kumar 2010: 118.

27 *BhaSū*$_1$: 2.10.10, p. 332. Glossary is as follows. Universe-space (*loyāgāsa*, Skt. *lokākāśa*), and non-universe-space (*aloyāgāsa*, Skt. *alokākāśa*).

28 *BhaSū*$_1$, 2.10.12, p. 334.

29 *BhaSū*$_4$, 11.10.11, p. 142; Schubring 1962: 204. Glossary is as follows. Hollow (*jhusira*, Skt. *suṣira*), and sphere (*gola*).

30 *BhaSū*$_1$: 2.1.24, p. 247. Glossary is as follows. Limited (*sa^aṃta*, Skt. *sānta*).

31 *BhaSū*$_3$: 8.10.29, p. 289. The canonical class of the Jaina school of Indian mathematics divides positive integers into three main divisions: numerable (*saṅkhyāta*), innumerable (*asaṅkhyāta*), and infinite (*ananta*). The lowest numerable number is 2 while the first innumerable number is the highest numerable number plus one and the first infinite number is the highest innumerable number plus one. This is the number-measure system of the Jainas. For its preliminary information, see Datta 1929: 140–142. And for unity not being a number, see Jadhav 2021*b*: 209–227.

universe, and the upper universe. The horizontal universe is in the shape of a round disc such as a kind of cymbal. The upper universe has the shape of an upright drum. It is divided into fifteen regions. Out of them, the first region is Sohammakappa and the fifteenth and uppermost region is Īsipabbhāra. The lower universe is divided into seven hells. Rayaṇappabhā is the first and uppermost hell, while the seventh and lower most hell is Ahesattamā.[32] The lower universe has the shape of a *tappa*.[33] "*Tappa* might be," writes Deleu, "*talpa*." The latter means bed in Sanskrit. The basis on which he writes so is that the universe is also described as having a bedstead-like shape in its lower part.[34] But Abhayadeva Sūri takes *tappa* as *tapra*. He also gives *uḍupaka*, which means boat in Sanskrit, as a synonym for *tapra*. He further explains that the lower universe is "shaped like an earthen bowl turned upside down."[35]

The horizontal universe rests on a disc the thickness of which is zero.[36] According to the *Bhagavatī Sūtra*, it is a flat plane divided into an innumerable number of concentric annuli which are alternatively islands and seas. The innermost region is known as the Jaṃbuddīva Island. Being the first annulus, it is a circular disc. The last annulus is the Sayaṃ-bhū-ramaṇa Sea.[37] At the very middle of the Jaṃbuddīva Island of diameter of 10^5 *yojana* is Mandara Mountain. It is 99×10^3 *yojana* high and 10^3 *yojana* deep. Its diameter at the foundation is $10090\frac{10}{11}$ *yojana*, at the surface is 10^4 *yojana* and at the top is 10^3 *yojana*.[38]

On the basis of these features it is very difficult to determine whether the universe as propounded in the *Bhagavatī Sūtra* is to be visualized as model 1a, 1b or 1c in Figure 1. The dimensions shown in each of the three figures are in *rajjus* where *rajju* is a linear measure. As far as the total volume of the universe is concerned, both the Śvetāmbara and Digambara Jaina thinkers have accepted that it is 343 cubic *rajjus* but their views on the exact shape of the universe differ.[39] According to the Digambara thinkers including Yativṛṣabha[40] (some date

32 *BhaSū*$_2$: 7.1.5, p. 328; *BhaSū*$_4$: 11.10.3–4 and 11.10.6–10, pp. 139–142; Deleu 1970: 176. Glossary is as follows. Firmly standing formation (*supa^iṭṭhaga saṃṭhāṇa*, Skt. *supratiṣṭhika saṃsthāna*), lower universe (*aholoya*, Skt. *adholoka*), horizontal universe (*tiriyaloya*, Skt. *tiryagloka*), upper universe (*uḍḍhloya*, Skt. *ūrdhvaloka*), cymbal (*jhallari*, Skt. *jhallaka*), upright drum (*uḍḍhamuiṃga*, Skt. *ūrdhvamṛdaṅga*), Sohammakappa (Skt. Saudharmakalpa), Īsipabbhāra (Skt. Īṣatprāgbhāra), and Ahesattamā (Skt. Adhaḥsaptamā).

33 *BhaSū*$_4$: 11.10.7, p. 141.

34 Deleu 1970: 177; *BhaSū*$_2$: 5.9.14, p. 160. Glossary is as follows. Bedstead (*paliyaṃka*, Skt. *palyaṅka*).

35 *BhaSū*$_8$: 395; also see Deleu 1970: 177. Glossary is as follows. Shaped like an earthen bowl turned upside down (*adhomukha-śarāvākāra-saṃsthāna*).

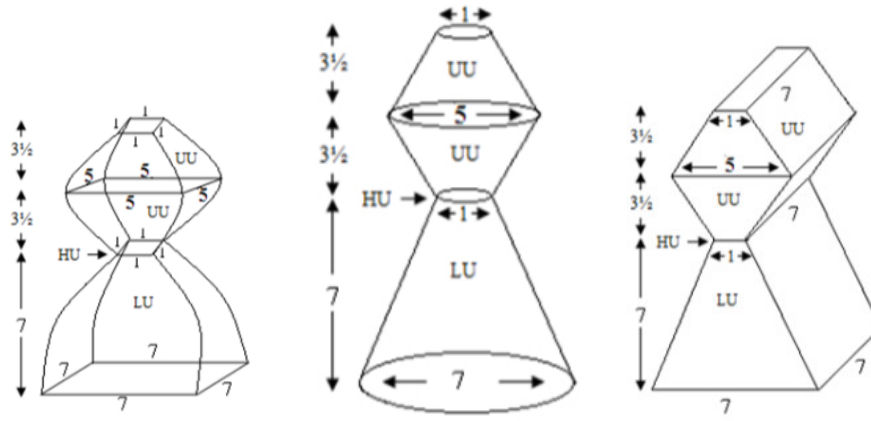
36 Schubring 1962: 216.

37 *BhaSū*$_4$: 11.10.5, p. 140. Glossary is as follows. Innumerable (*asaṃkhejja*, Skt. *asaṅkhyeya*), and Sayaṃ-bhū-ramaṇa (Skt. *svayam-bhū-ramaṇa*, "Self-born merriment").

38 *JaDvīPraSū*: 4.132, pp. 361–362.

39 Kumar 2010: 144–145.

40 *TiLoPa*: vv. 1.136–138, pp. 32–33 and vv. 148–149, pp. 37–38.



(a) (b) (c)

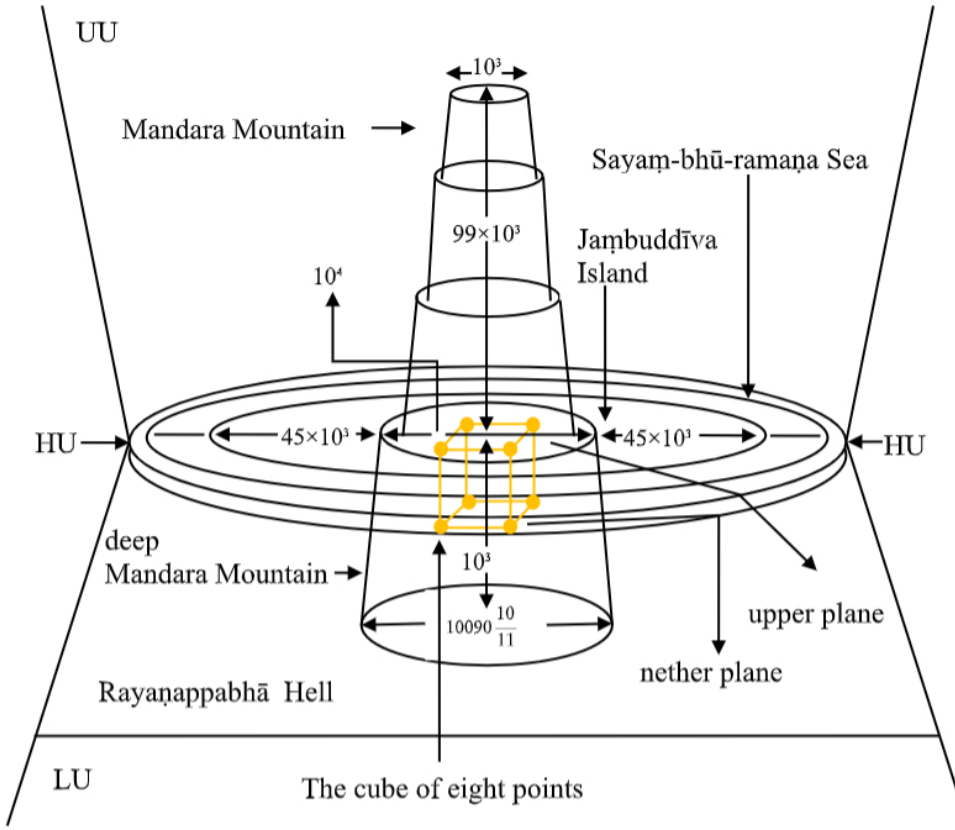
Figure 1: Three possible models of the universe.

between 176 CE and 609 CE), Vīrasena[41] (c. 816 CE), and Nemicandra[42] (c. 981 CE), the universe is in the form of three trapezoidal prisms piled one upon another as shown in Figure 1 as model 1c. Model 1b suggests that the universe is in the form of three truncated cones placed one upon another. But Vīrasena refers to that it is not the shape of the universe as its volume comes to be nearly 161 cubic *rajjus*, which is less than half of the accepted volume of the universe.[43] The shape of the universe as propounded in the *Bhagavatī Sūtra* might be model 1a as Muni Mahendra Kumar claims to have reached it while retaining all the fundamental beliefs and dimensions intact.[44] But the dimension and shape of the horizontal universe appears to fit model 1b. This is because the Sayaṃ-bhū-ramaṇa Sea is the last circular annulus on the horizontal universe.

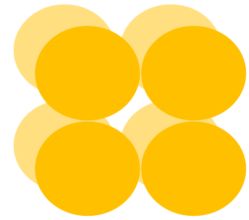
ILLUSTRATION OF THE POSITION OF THE CUBE OF EIGHT SPACE-POINTS

Based on the above details we are able to illustrate where the cube of eight space-points is, although it is an issue which of the models in Figure 1 is the model of the universe according to the *Bhagavatī Sūtra*. The lower four points of the cube lie in the nether plane and its upper four points lie in the upper plane. These two planes are adjacent. See Figure 2a.[45] Those eight points are interlinked in such a way that each point is adjacent to the other seven points; to the three points

41 *ṢaKhaĀ*: 1.3.2, pp. 19–22.
42 *TriLoSā*: v. 113, p. 110.
43 *ṢaKhaĀ*: 1.3.2, vv. 6–7, pp. 11 and 13.
44 Kumar 2010: 155–159, 388–402.
45 For plain drawing of the cube of eight space-points, see *BhaSū*$_8$: 562. For the figure drawn by Trilok Sharma to illustrate it, see page just after p. 4 in *BhaSū*$_4$.



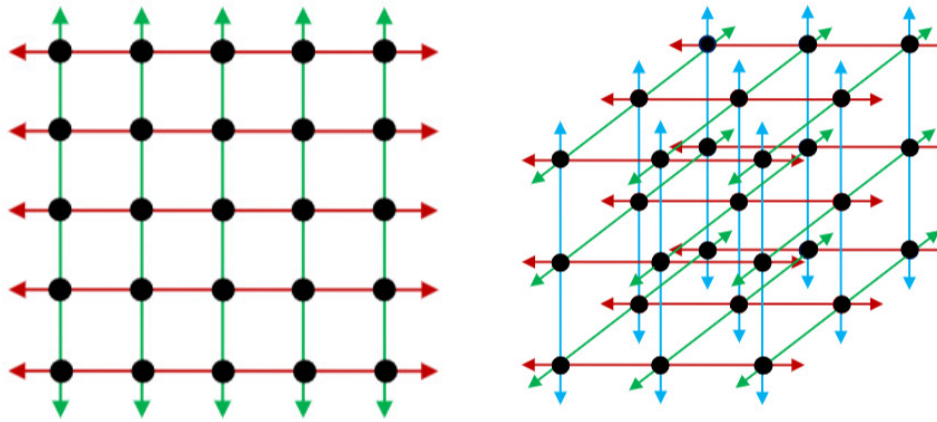
(a) The cube of eight points.



(b) Close-up view.

Figure 2: The cube of eight points.

forming the edges of the cube, to the three points forming the diagonals of the surfaces of the cube, and to one point forming the diagonal of the cube. This is why they form the smallest cube. The adjacency of those eight points can be better understood through Figure 2b.



(a) Horizontal. (b) Horizontal and vertical.

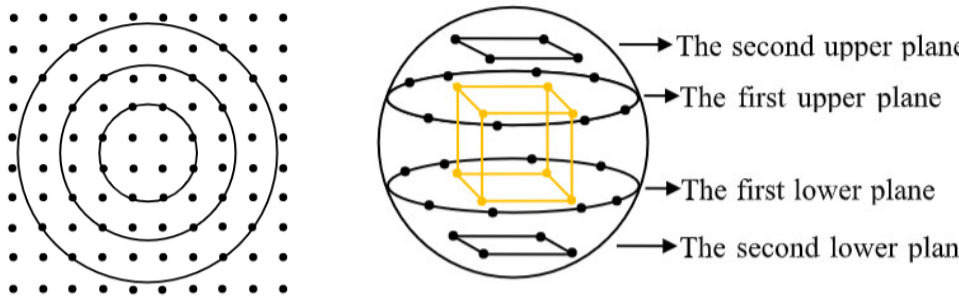
Figure 3: System of points as described in the *Bhagavatī Sūtra*.

SYSTEM OF POINTS IN THE BHAGAVATĪ SŪTRA

In chapter twenty-five of the *Bhagavatī Sūtra*, the term used for line is *seḍhi*. While commenting on the term, Abhayadeva Sūri writes that *seḍhi* means a long row and here it is acceptable as a row of space-points.[46] It is further stated in the same chapter that lines in general are infinite in substance whether they run from east to west or from south to north or from zenith to nadir. This is also the case with the lines in the non-universe. But the lines in the universe are innumerable. This is, Abhayadeva says, because there are innumerable points in the universe. As in substance, lines usually are infinite in length in whatever direction they run. In the universe, the horizontal lines, i.e., the lines running from east to west and from north to south are numerable or innumerable in length while the vertical lines are innumerable in length. In the non-universe, the horizontal lines are infinite in length while the vertical lines, i.e., the lines running from nadir to zenith are infinite, innumerable or numerable in length. Lines as a whole have no beginning and no end in whatever direction they run. All lines in the universe have a beginning and an end. In the non-universe, the vertical lines may have a beginning and an end and both the vertical and horizontal lines may have neither of these or one of these.[47]

46 *BhaSū*$_9$: 25.3, pp. 414–415. Glossary is as follows. *Seḍhi* (Skt. *śreṇī*), row (*paṅkti*), and space-points (*ākāśapradeśas*).

47 *BhaSū*$_9$: 25.3, pp. 414–419; Deleu 1970: 273. Glossary is as follows. Substance (i.e., number, *davva*, Skt. *dravya*), infinite (*aṇaṃta*, Skt. *ananta*), east to west (*pāīṇapaḍīṇa*, Skt. *pūrva-paścima*), south to north (*dāhiṇuttara*, Skt. *dakṣiṇa-uttara*), zenith to nadir (*uḍḍhamaha*, Skt. *ūrdhva-adhaḥ*), lines in the non-universe



(a) (b)

Figure 4: Constructing a core-circle and a core-sphere.

Based on the above, we can say that in the *Bhagavatī Sūtra* such a system of points is considered that the points lie in a plane in a rectangular system of their horizontal rows as shown in Figure 3a and they are located in space in a rectangular system of their horizontal and vertical rows as shown in Figure 3b.[48] This interpretation will serve as a basic framework for us to understand the figurate numbers given in it. The above characteristics of the lines in the non-universe confirm that the non-universe is spherical in shape. Every line whether it is horizontal or vertical is a chord of the circular cross-section of the spherical non-universe.

CORE-CIRCLE AND CORE-SPHERE

In the *Bhagavatī Sūtra*, the circle is constructed by means of points in two ways. In way one the centre of a circle is determined by a single point and in way two it is determined by the core of the four points forming a square. For this reason I used the terms centered-circle and core-circle. Similar is the case with sphere. The number of points required to construct a centered-circle and a centered-sphere is always odd.[49] On the other hand, the number of points required to construct a core-circle and a core-sphere is always even. Here, it is relevant to see how the Jainas arrived at these even numbers. The way must have been as follows.[50]

We first set four points on the plane as a core forming a square. From then on, keeping the same distance in each of the two consecutive circles, we draw

(*alogāgāsaseḍhīs*, Skt. *aloka-ākāśa-śreṇis*), lines in the universe (*logāgāsaseḍhīs*, Skt. *loka-ākāśa-śreṇis*), length (i.e., region, *paesa*, Skt. *pradeśa*), numerable (*saṃkhejja*, Skt. *saṅkhyeya*), beginning (*sāiya*), no beginning (*aṇāiya*), end (*sapajjavasiya*, Skt. *saparyavasita*, i.e., *sānta*), and no end (*apajjavasiya*, Skt. *aparyavasita*, i.e., *ananta*).

48 I suggested Figure 3a and Figure 3b for the first time in my paper published in 2009. See Jadhav 2009: 40.

49 Jadhav 2009: 46–48.

50 Jadhav 2009: 48–50.

con-core-circles. The circumference of each such circle goes through eight points. This means that 12 points are required to form the foremost core-circle including its disc. We are able to see the core-circles up to the fourth one in Figure 4a.

The expression for the total number of points required to construct the n^{th} core-circle, say $C(n)$, can, if we apply inductive reasoning, be given by,[51]

$$C(n) = \begin{cases} 4 & \text{when } n = 1 \\ (2n)^2 - 4 & \text{when } n \geq 2. \end{cases}$$

The number of points required to construct the core-circles are

$$4, 12, 32, 60, 96, \ldots$$

when $n = 1, 2, 3, \ldots$ respectively.

Looking at this series, we are able to point out some of its salient features. The first term, i.e., the four points, does not form a core-circle. In fact, they form the core of the four points forming a square. But by convention this square is the first core-circle. The number of points required to construct a core-circle is always even. Since the first term of the series does not form a core-circle, the least even number of points required to construct a core-circle is 12.

If the least core of a core-circle is a square of four points, then that of a core-sphere will be a cube of eight points.[52] The upper four points of the cube are located in the first upper plane and its lower four points are located in the first lower plane. By convention, this cube will be the first core-sphere. It is very interesting to note that this cube will always remain the least core of any bigger core-sphere, no matter how big the core-sphere is. The second sphere is comprised of the points located in the four planes. It contains the second core-circle two times, one being in the first upper plane and the other being in the first lower plane and the first core-circle two times, one being in the second upper plane and the other being in the second lower plane. See Figure 4b. The third core-sphere is formed by placing the upper hemispherical part of the second core-sphere atop one third core-circle located in the first upper plane and the lower hemispherical part of the second core-sphere below the other third core-circle located in the first lower plane; this way the third core-sphere occupies its points in the six planes.

51 I did not suggest any expression in my paper published in 2009 for the total number of points required to construct the n^{th} core-circle. See Jadhav 2009: 48. In this paper I suggest it.

52 This reasoning is not mine. I have extracted it from the information given in the *Bhagavatī Sūtra* that the minimum even number of points required to form core-circle and core-sphere are 12 and 32 respectively. See Jadhav 2009: 50.

Using inductive reasoning the expression for the total points required to construct the n^{th} core-sphere, say $S(n)$, can be given by,[53]

$$S(n) = \frac{4}{3}(2n^3 + 3n^2 - 5n + 6)$$

The number of points required to construct the core-spheres are

$$8, 32, 96, 216, ...$$

when $n = 1, 2, 3, ...$ respectively.
Looking at this series, we are able to point out that the first term, i.e., the eight points, does not form a core-sphere. In fact, they form the core of eight points forming a cube. But by convention, this cube is the first core-sphere. The number of points required to construct a core-sphere is always even. Since the first term of the series does not form a core-sphere, the least even number of points required to construct a core-sphere is 32.

UNDERSTANDING THE CUBE OF EIGHT POINTS

Now we are able to understand how the cube of eight points relates to the horizontal universe and the non-universe. The upper and nether thin planes of the Rayaṇappabhā hell are adjacent. The upper thin plane is the horizontal universe. It expands into the non-universe and becomes the first upper plane of the non-universe. The same is the case with the nether thin plane, which expands into the non-universe and becomes the first lower plane of the non-universe. Since the non-universe is a hollow sphere, these two planes must be its biggest equal and parallel core-circles, say $C(n)$ where n approaches infinity. Since the horizontal universe is a core-circle, the upper four points of the cube of eight points, forming a square, are its core in a true mathematical sense. See Figures 2a and 4b to visualize this. Since the non-universe is a sphere, the cube of eight points is its core.

UNDERSTANDING THE TEN DIRECTIONS IN MATHEMATICAL TERMS

In chapter ten of the *Bhagavatī Sūtra* it is stated that east, west, south, north, up, and down are the six main directions. It is further stated that there are actually ten directions. With their proper names they are:[54]

53 I did not suggest any expression in my paper published in 2009 for the total number of points required to form the n^{th} core-sphere. See Jadhav 2009: 50. In this paper I suggest it. For the sake of explanation, I write that $S(n) = C(1) + C(2) + ... + C(n-1) + C(n) + C(n) + C(n-1) + ... + C(2) + C(1)$.

54 *BhaSū*$_5$: 10.1.2–5, pp. 477–479; *BhaSū*$_4$: 10.1.2–6, pp. 2–4. Glossary is as follows. East (*pāīṇā*, Skt. *pracīnā*), east

i east – *Iṃdā*,
ii east-south – *Aggeyī*,
iii south – *Jammā*,
iv south-west – *Neraī*,
v west – *Vāruṇī*,
vi west-north – *Vāyavvā*,
vii north – *Somā*,
viii north-east – *Īsāṇī*,
ix zenith – *Vimalā*, and
x nadir – *Tamā*.

In section two of this paper, we have noticed from the passage of the *Bhagavatī Sūtra* that the above ten directions originated from the cube of eight points. The same is given in chapter ten of the *Sthānāṅga Sūtra*.[55]

It is further stated in the *Bhagavatī Sūtra* that the east direction starts with two points. It increases by two points at each further step. It is shaped like a drum in the universe and like the part of a carriage in the non-universe. Similar is the case with the other three cardinal directions, namely, south, west, and north. The east-south direction starts with one point. It extends by adding another point to each further step without any increase. It is shaped like a broken string of pearls. Similar is the case with the other three intermediate directions, namely, south-west, west-north, and north-east. The zenith direction starts with four points. It extends by adding another four points to each subsequent step without increment. It is in the shape of a cuboidal column. Similar is the case with the nadir direction. Every direction whether cardinal or intermediate or up or down has its origination and flow from the cube of eight points and occupies innumerable points in the universe and infinite points in the non-universe and has a beginning and an end in the universe and a beginning but not an end in the non-universe.[56]

On the basis of the above features, these ten directions can be better understood through Figure 5, which shows how they start from the cube. It can be deduced that the intermediate directions are linear while the cardinal directions

(*puratthimā*, Skt. *paurastyā*), west (*paḍīṇā*, Skt. *praticīnā*), west (*paccatthimā*, Skt. *pāścātyā*), south (*dāhiṇā*, Skt. *dakṣinā*), north (*udīṇā*, Skt. *udīcīnā*), north (*uttarā*), up (*uḍḍhā*, Skt. *urdhvā*), down (*aho*, Skt. *adhaḥ*), *Iṃdā* (Skt. *Indrā*), east-south (*puratthimdāhiṇā*, Skt. *paurastyadakṣiṇā*), *Aggeyī* (Skt. *Āgneyī*), *Jammā* (Skt. *Yāmyā*), south-west (*dāhiṇpaccatthimā*, Skt. *dakṣiṇapāścātyā*), *Neraī* (Skt. *Nerṛtī*), west-north (*paccatthimuttarā*, Skt. *pāścātyottarā*), *Vāyavvā* (Skt. *Vāyavyā*), *Somā* (Skt. *Saumyā*), north-east (*uttarapuratthimā*, Skt. *uttarapaurastyā*), and *Īsāṇī* (Skt. *Ēśānī*).

55 *SthāSū*$_1$: chapter 10, under sūtra 720, v. 153, p. 823.

56 *BhaSū*$_6$: 13.4.14–16, pp. 45–46. Also see Deleu 1970: 195. Glossary is as follows. Drum (*muraja*), part (*uddhi*), carriage (*sagaḍa*, Skt. *śakaṭa*), cardinal directions (*disās*, Skt. *diśās*), "broken string of pearls" (*chiṇṇamuttāvalī*, Skt. *chinnamuktāvali*), intermediate directions (*vidisās*, Skt. *vidiśās*), and cuboidal (*ruyaga*, Skt. *rucaka*). 'Broken string of pearls' has been a motif in Indian poetry and mathematics. It is also known as 'broken string and scattered pearls.' For details on the motif, see Sarma 2004: 463–476. Also see Jadhav 2021*a*: 104–109.

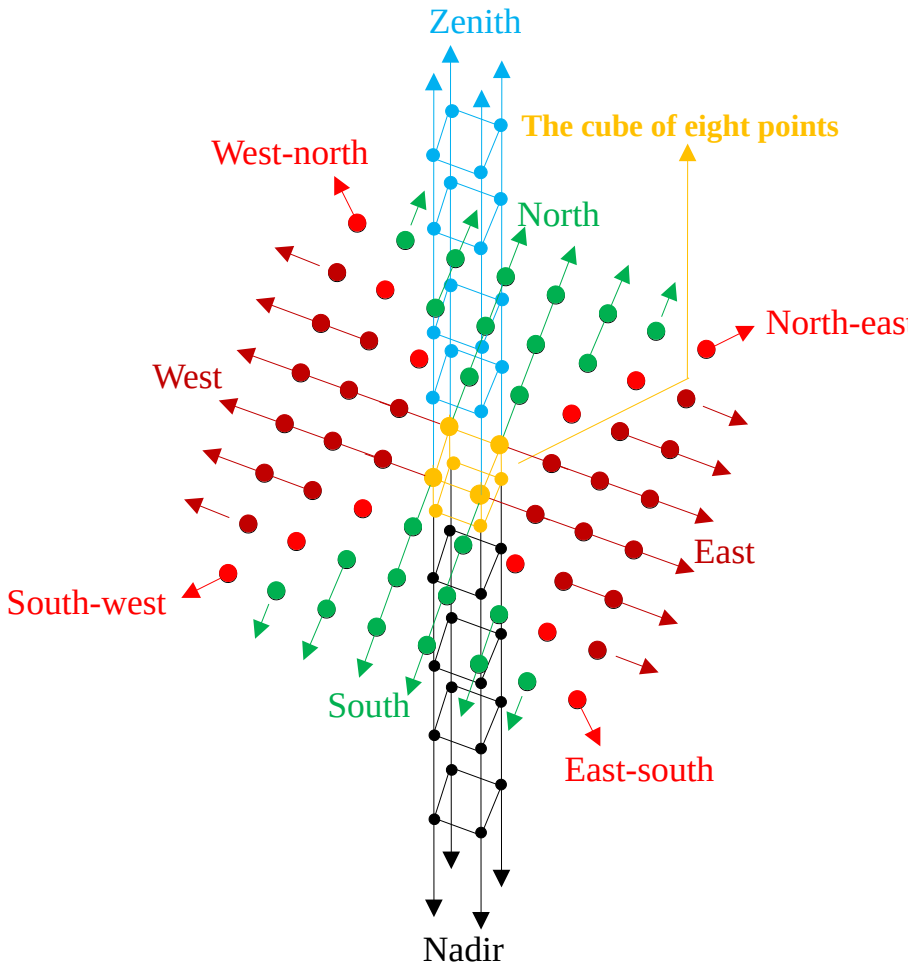


Figure 5: The ten directions.

are planar and the up and down directions are three dimensional.[57] We have noticed above that the cardinal direction is shaped in the universe like a drum and in the non-universe like the part of a carriage. The part of a carriage, according to Deleu's interpretation, is its seat.[58] It seems to me to be the full seat of an old-fashioned bullock cart. In fact, both the drum-shaped cardinal direction in the universe and the carriage-seat-shaped cardinal direction in the non-universe

57 A figure drawn by Trilok Sharma to illustrate the ten directions shows that both the cardinal and intermediate directions are three dimensional, although its description does not read so at all. See two pages between pp. 4 and 5 in *BhaSū*$_4$.

58 Deleu 1970: 195.

are isosceles trapezia. The smaller parallel side of the drum-shaped cardinal direction consists of two points while that of the carriage-seat-shaped cardinal direction innumerable points.

4 CONCLUDING REMARKS

In this paper, by exploring the mathematical approach on which the cube of eight points in Jain cosmology was based, we suggest the following as a plausible explanation to understand it. The core of the non-universe is the core of a core-sphere when the core-sphere is formed using n core-circles such that they are stacked up and down in the same order from the n^{th} core-circle to the first core-circle. The upper four points of the cube of eight points form the core of the horizontal universe and the cube of eight points is the core of the non-universe. An n^{th} core-circle becomes the horizontal universe when n is innumerable and an n^{th} core-sphere forms the non-universe when n approaches infinity.

HISTORICAL IMPORTANCE

Many interpretations including cosmographical and mathematical ones, can be drawn from the above results and discussion. From the very beginning I have confined myself to their mathematical interpretation and would like to extend it to the history of figurate numbers. It is of great mathematical interest and historical importance to us that the models of both the horizontal universe and the non-universe were originally developed by the Jaina thinkers with the involvement of the concept of figurate numbers. Otherwise, they would not have referred to that there is the core of the horizontal universe where a cube of eight points is and ten directions originate from those eight points.

The Greek mathematicians such as Pythagoras (572–510 BCE), Philolaus (480–385 BCE), Nicomachus (c. 100 CE), and Theon of Smyrna (c. 125 CE) paid great attention to the development of figurate numbers. They counted figurate numbers by arranging dots or pebbles in the shapes of different geometrical forms. On the other hand, the figurate numbers in the *Bhagavatī Sūtra* were counted, before or certainly during the period from the fourth century BCE to the fifth century CE, by joining the points located in a plane or space in a rectangular system. The Greeks discussed plane numbers such as triangular numbers, square numbers, oblong numbers, pentagonal numbers, hexagonal numbers, etc., and solid numbers such as cube numbers, pyramidal numbers, etc.[59] We do not have any report which reads that circle numbers (i.e., centered-circle numbers and core-circle numbers), spherical numbers (i.e., centered-spherical

59 Heath 1921: 76–84; Hollingdale 1989: 15–16; Boyer 1968: 59–60; Smith 1958: 24–25; Eves 1969: 56–57; Katz 2009: 37–38, 173–175.

numbers and core-spherical numbers), outer-lying-circular-annulus numbers, and outer-lying cylindrical numbers were included in their discussion or not except that David Eugene Smith refers to that Nicomachus included spherical numbers in his discussion. Smith gives us this information from the *Boetti de institutione arithmetica libri duo* of Boethius (died 524 CE).[60] Not only core-circle numbers and core-spherical numbers relate to the Jaina models of the horizontal universe and the non-universe but also outer-lying-circular-annulus numbers and outer-lying-cylindrical numbers pertain to them. If every number of the series of core-circle numbers is subtracted from its succeeding term starting from 12, we get the series of core-circular-annulus numbers, i.e., 8, 20, 28, 36, ... Since the first term of this series forms the circumference of the first core-circle, in fact, of the second core-circle if the core is considered to be the first core-circle, the least even number of points required to construct outer-lying-circular-annulus is 20, which is given in the *Bhagavatī Sūtra*.[61] Every term of the series of outer-lying-circular-annulus numbers forms its own series of outer-lying-circular-cylindrical numbers when the corresponding outer-lying-circular annulus is stacked on itself. The first few outer-lying-cylindrical numbers are 16, 24, 32, 40, ...; 40, 60, 80, 100, ...; 56, 84, 112, 140, ...; ... Since the points in the first series form no more than the curved surfaces of every first circular cylinder, the least number of points required to construct an outer-lying-circular-cylinder is 40, which is given in the *Bhagavatī Sūtra*.[62] In Figure 2a, we are able to see the outer-lying-circular annuli in the horizontal universe and to visualize the outer-lying-circular cylinders formed in the parallel planes starting downward from the horizontal universe.[63]

The systems of points, both two dimensional (Figure 3a) and three dimensional (Figure 3b), culled from the *Bhagavatī Sūtra* is significant in the sense that by way of them the formations of isosceles triangle, pyramid of isosceles triangular base, square, cube, rectangle, line, cuboid, circle, sphere, outer-lying circular annulus, outer-lying circular cylinder, the core of the non-universe as a cube of eight points, and the ten directions with their shapes and patterns of points were conceived in India by the Jaina thinkers. As a result, their details were recorded in the *Bhagavatī Sūtra* with reference to the minimum points.

TRANSMISSION

If the period of the *Bhagavatī Sūtra* is compared with that of Pythagoras and Philolaus, it may be suspected that the ideas of figurate numbers, especially of line, isosceles triangle, square, rectangle, pyramid of isosceles triangular base,

60 Smith 1958: 25; *BDIALD*: 104 and 121.
61 *BhaSū*$_9$: 25.3.726, p. 407; Jadhav 2009: 50–51.
62 *BhaSū*$_9$: 25.3.726, p. 407; Jadhav 2009: 51.
63 For exact illustrations, see Jadhav 2009: 51.

cube, and cuboid, might have been transmitted from Greece to India and those of the rest were developed by the Jaina thinkers in accordance with their requirements. But the lack of concrete evidence of transmission such as Greek loanwords related to figurate numbers in any Prakrit (or Sanskrit) treatise must surely lead us to conclude that such transmission did not occur and that the ideas of figurate numbers arose independently in the Greek and Jaina cultures.

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ABBREVIATIONS AND NOTATION

HU	Horizontal universe.
LU	Lower universe.
Skt.	Sanskrit. The term(s) added by me after this abbreviation are intended to aid the reader to understand Prakrit terms through their Sanskrit equivalents.
UU	Upper universe.
[...]	A pair of square brackets, wherever used, contains a paraphrase inserted by me to achieve comprehensiveness together with clarity. It does not mean that the original expressions are incomplete or corrupted.

PRIMARY TEXTS

BDIALD Friedlein, Godofredus (1867) (ed.), *Anicii Manlii Torquati Severini Boetii. De institutione arithmetica libri duo; De institutione musica libri quinque; accedit geometria quae fertur Boetti* (Lipsiae: Teubner), ARK: ark:/13960/t9475gh29.

BhaSū$_1$ Amar Muni and 'Saras', Srichand Surana (2005) (eds.), **सचित्र श्री भगवती सूत्र** = *Illustrated Shri Bhagavati Sutra* (*Vyakhya Pra-*

jnapti), *Volume I, Chapters 1–4: Original Text with Hindi and English Translations, Elaboration and Multicoloured Illustrations*, trans. S. Bothara (Delhi: Padma Prakashan), https://jainelibrary.org/book-detail/?srno=002902, (on 6 Oct. 2023).

BhaSū$_2$ Amar Muni and 'Saras', Srichand Surana (2006*a*) (eds.), *Illustrated Shri Bhagavati Sutra* (*Vyakhya Prajnapti*), *Volume II, Chapters 5–8,* (*Original Text with Hindi and English Translations, Elaboration and Multicoloured Illustrations*), trans. S. Bothara (Delhi: Padma Prakashan), https://jainelibrary.org/book-detail/?srno=002903, (on 11 Apr. 2021).

BhaSū$_3$ Amar Muni, Varun Muni, 'Saras', Srichand Surana, and Surana, Sanjay (2008) (eds.), **सचित्र श्री भगवती सूत्र** = *Illustrated Shri Bhagavati Sutra* (*Vyakhya Prajnapti*), *Volume III, Chapters 8–9: Original Text with Hindi and English Translations, Elaboration and Multicoloured Illustrations*, trans. S. Bothara (Delhi: Padma Prakashan), https://jainelibrary.org/book-detail/?srno=002904, (on 11 Apr. 2021).

BhaSū$_4$ Amar Muni, Varun Muni, and Surana, Sanjay (2013) (eds.), **सचित्र श्री भगवती सूत्र** = *Illustrated Shri Bhagavati Sutra* (*Vyakhya Prajnapti*), *Volume IV, Chapters 10–13: Original Text with Hindi and English Translations, Elaboration and Multicoloured Illustrations*, trans. S. Bothara (Delhi: Padma Prakashan), https://jainelibrary.org/book-detail/?srno=002493, (on 11 Apr. 2021).

BhaSū$_5$ Līlamabāī, Āratībāī, and Subodhikābāī (2009*a*) (eds.), **श्री भगवती सूत्र (मूल पाठ,भावार्थ, विवेचन, परिशिष्ट** [= *Śrī Bhagavatī Sūtra with Original Text, Translation, Annotations, and Appendices*] *Volume III, Chapters 8-12*, trans. Āratībāī (Mumbai: Śrī Guru Prāṇa Prakāśana), https://jainelibrary.org/book-detail/?srno=008760, (on 6 Oct. 2023).

BhaSū$_6$ Līlamabāī, Āratībāī, and Subodhikābāī (2009*b*) (eds.), **श्री भगवती सूत्र (मूल पाठ,भावार्थ, विवेचन, परिशिष्ट** [= *Śrī Bhagavatī Sūtra with Original Text, Translation, Annotations, and Appendices*] *Volume IV, Chapters 13–23*] (Mumbai: Śrī Guru Prāṇa Prakāśana), https://jainelibrary.org/book-detail/?srno=008761, (on 6 Oct. 2023).

BhaSū$_7$ Mahāprajña, Ācārya (2007*a*) (ed.), **भगवई (विआहपण्णत्ती) (मूल पाठ, संस्कृत छाया, हिन्दी अनुवाद, भाष्य, तथा अभयदेवसूरिकृत वृत्ति एवं परिशिष्ट—शब्दानुक्रम आदि सहित)** = *Bhagawaī* (*Viāhapaṇṇattī*) *Volume IV, Sataka 12–16* (*Prakrit Text, Sanskrit Renderings, Hindi Translation and Critical Annotations, with Vṛtti of Abhayadevasūri and Appendices-Indices*

etc. (Ladnun: Jain Vishva Bharati), https://jainelibrary.org/book-detail/?srno=003596, (on 6 Oct. 2023).

BhaSū$_8$ श्रीमदभयदेवसूरिसूत्रितविवरणयुतं श्रीमद्भगवतीसूत्रं (द्वितीयो विभागः) = *Śrī Bhagavatī Sūtra with the Commentary of Abhayadeva Sūri Volume II* (1919) (Mehesana: Agamodayasamiti), https://jainelibrary.org/book-detail/?srno=600225, (on 6 Oct. 2023); references are to PDF page numbers.

BhaSū$_9$ श्रीमद्...अभयदेवसूरिप्रणीतवृत्तियुतं श्रीमद्भगवतीसूत्रं (तृतीयो विभागः) = *Śrī Bhagavatī Sūtra with the Commentary of Abhayadeva Sūri Volume III* (1921) (Mehesana: Agamodayasamiti), https://jainelibrary.org/book-detail/?srno=600226, (on 6 Oct. 2023); references are to PDF page numbers.

JaDvīPraSū Amar Muni and 'Saras', Srichanda Surana (2006*b*) (eds.), *Illustrated Jambudveep Prajnapti Sutra: with Basic Prakrit Text: Hindi-English Translation Alongwith Elaboration and Illustrations*, trans. R. K Jain (Delhi: Padma Prakashan), https://jainelibrary.org/book-detail/?srno=002911, (on 6 Oct. 2023).

ṢaKhaĀ Jain, Hīralāla, Pūlacandra Siddhāntaśāstrī, Bālacandra Siddhāntaśāstrī, Devakīnandana Siddhāntaśāstrī, and Upadhye, A. N. (1942) (eds.), श्री–भगवत्–पुष्पदन्त–भूतबलि–प्रणीतः षट्खंडागम श्रीवीरसेनाचार्य–विरचित–धवला–टीका–समन्वितः तस्य चतर्थखंडे वेदनानमधेये ...कृतिअनुयोगद्वाम: [= *Ṣaṭkhaṇḍāgama of Puṣpadanta and Bhūtabalī with the Dhavalā Commentary of Vīrasena*], trans. H. L. Jain et al. (reprint, Solapur 2005, Amarāvatī: Śrimanta Seṭha Śitābarāya Lakṣmīcandra), https://jainelibrary.org/book-detail/?srno=001403, (on 15 Apr. 2021).

SthāSū$_1$ Jambūvijaya Muni (2003) (ed.), *Sthānāṅga Sūtra, Part III. With the commentary of Abhayadeva Sūri* (Mumba: Śrī Mahāvīra Jaina Vidyālaya), https://jainelibrary.org/book-detail/?srno=001029, (on 6 Oct. 2023).

SthāSū$_2$ Amar Muni and Surana, Srichand (2004) (eds.), *Illustrated Shri Sthaananga SutraPart II, (Original Text with Hindi and English Translations, Elaboration and Multicoloured Illustrations)*, (Delhi: Padma Prakashan), https://jainelibrary.org/book-detail/?srno=002906, (on 6 Oct. 2023).

TiLoPa Patni, Cetanaprakāśa (1997) (ed.), श्रीयतिवृषभाचार्यविरचिता तिलोय-पण्णत्ती (त्रिलोकप्रज्ञप्तिः). प्रथम खण्ड भाषाटीका आर्यिका श्री विशुद्धमति माताजी: [= *Tiloyapaṇṇatti of Yativṛṣabha with Āryikā Viśuddhamati's Hindi commentary*], v. 1 (Dehra-Tijara: Śrī 1008 Candraprabha Digambara

Jaina Atiśayakṣetra), https://jainelibrary.org/book-detail/?srno=090504, (on 6 Oct. 2023).

TriLoSā Mukhtāra, R. C. Jain, and Pāṭanī, Cetana Prakāśa (1975) (eds.), श्रीमन्नेमिचन्द्र सिद्धान्तचक्रवर्ति विरचित त्रिलोकसार (श्रीमन्माधवचन्द्रत्रैविद्यदेवकृत व्याख्या सहित) हिन्दी टीकाकर्त्री आर्यिका श्री विशुद्धमति माताजी [=*Trilokasāra of Nemicandra with Mādhavacandra Traividya's Sanskrit Commentary and Āryikā Viśuddhamati's Hindi Commentary*] (Śrī Śivasāgara Granthamālā; Śrī Mahāvīrajī (Rājasthāna): Lāḍamala Jaina), https://jainelibrary.org/book-detail/?srno=090512, (on 16 Apr. 2021).

REFERENCES

Boyer, C. B. (1968), *A History of Mathematics* (New York: John Wiley & Sons, Inc.), ARK: ark:/13960/t71w45665.

Datta, B. B. (1929), "The Jaina School of Mathematics," *Bulletin of the Calcutta Mathematical Society*, 21: 115–45, ARK: ark:/13960/t63540n04.

Deleu, Jozef (1970), *Viyāhapannatti* (*Bhagavai*). *The Fifth Anga of the Jaina Canon. Introduction, Critical Analysis, Commentary & Indexes* (Brugge: De Tempel), ARK: ark:/13960/t25b5960n.

Eves, Howard W. (1969), *Introduction to the History of Mathematics* (3rd edn., New York: Holt, Rinehart and Winston), ARK: ark:/13960/s215xws03r4.

Gelra, M. R. (1999), "Concept of the Eight Point Centre in Jaina Cosmology: a Critique," तुलसी प्रज्ञा = *Tulsī Prajñā*, 26/106: 121–30, https://jainelibrary.org/book-detail/?srno=524599, (on 27 Sept. 2023).

Heath, T. (1921), *A History of Greek Mathematics: From Thales to Euclid* (Oxford: The Clarendon Press).

Hollingdale, S. (1989), *Makers of Mathematics* (London: Penguin Books).

Jadhav, Dipak (2009), "On the Figurate Numbers from the Bhagavatī Sūtra," *Gaṇita Bhāratī*, 31/1–2: 35–55.

—— (2017), "The Jaina School of Indian Mathematics," *IJHS* 52/3: 316–34. DOI: 10.16943/ijhs/2017/v52i3/49161.

—— (2021*a*), "In How Many Days Will He Meet His Wife?," *Journal of Humanistic Mathematics*, 11/1: 95–112. DOI: 10.5642/jhummath.202101.07.

—— (2021*b*), "Jaina Thoughts on Unity Not Being a Number," *History of Science in South Asia*, 9 (Aug.): 209–31. DOI: 10.18732/hssa67.

—— (2022), "Partitions of Numbers Described in Bhagavatī Sūtra," in S. G. Dani and S. K. Jain (eds.), *Mathematics in Ancient Jaina Literature* (Singapore: World Scientific Publishing), 87–105.

Jain, S. K. (1975), "Chronology of Ancient Jaina Conferences," *The Jaina Antiquary*, 27/1: 31–4.

Katz, Victor J. (2009), *A History of Mathematics: An Introduction* (3rd edn., Reading, Mass., Harrow, England, etc.: Addison-Wesley), ISBN: 0-321-01618-1, https://edisciplinas.usp.br/mod/resource/view.php?id=3470017.

Krümpelmann, Kornelius (2006), "The Sthānāṅgasūtra – an Encyclopaedic Text of the Śvetāmbara Canon," *International Journal of Jaina Studies*, 2/2: 1–13. DOI: 10.5281/zenodo.8417441.

Kumar, Muni Mahendra (2010), *The Enigma of the Universe* (Ladnun: Jain Vishva Bharati University).

Pragya, Samani Chaitanya (2005), *Scientific Vision of Lord Mahāvīra:* (*With Special Reference to Bhagavatī Sūtra*) (Ladnun: Jain Vishva Bharati), ARK: ark:/13960/s2r9nj2c5rs.

Rajgopal, P. (1991), "The Sthānāṅga Sūtra Programme in Indian Mathematics," *Arhat Vacana*, 3/2: 1–8.

Sarma, Sreeramula Rajeswara (2004), "Broken String – Scattered Pearls: A Motif in Sanskrit Poetry and Mathematics," *Studia Asiatica*, V–VI, ed. E. Ciurtin: 463–76. DOI: 10.5281/zenodo.8417350.

Schubring, Walther (1962), *The Doctrine of the Jainas. Described After the Old Sources* (Delhi: Motilal Banarsidass), ARK: ark:/13960/t6257115q; first published in German, 1934; translated into English by Wolfgang Beurlen.

Shah, R. S. (2008), "Mathematical Ideas in Bhagavatī Sūtra," *Gaṇita Bhāratī*, 30/1: 1–25.

Sikdar, Jogendra Chandra (1964), *Studies in the Bhagawatīsūtra*, ed. Nathmal Tatia (Muzaffarpur: Research Institute of Prakrit, Jainalogy & Ahimsa), https://jainelibrary.org/book-detail/?srno=001615, (on 27 Sept. 2023).

Smith, D. E. (1958), *A History of Mathematics. Vol. II: Special Topics of Elementary Mathematics* (2nd edn., New York: Dover), ARK: ark:/13960/t22b8w241.

Wiles, Royce (2006), "The Dating of the Jain Councils: Do Scholarly Presentations Reflect the Traditional Sources?," in Peter Flügel (ed.), *Studies in Jaina History and Culture: Disputes and Dialogues* (London: Routledge), chap. 3, 61–85, ISBN: 9780203008539. DOI: 10.4324/9780203008539-12.

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